

What can we learn about neutron stars from gravity-wave observations?*

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In the next few years, the first detections of gravity-wave signals using Earth-based interferometric detectors will begin to provide precious new information about the structure and dynamics of compact bodies such as neutron stars. The intrinsic weakness of gravity-wave signals requires a proactive approach to modeling the prospective sources and anticipating the shape of the signals that we seek to detect. Full-blown 3-D numerical simulations of the sources are playing and will play an important role in planning the gravity-wave data-analysis effort. I review some recent analytical and numerical work on neutron stars as sources of gravity waves.

I. INTRODUCTION

In the course of the next decade, the inception of gravity-wave (GW) astronomy will open an exciting new window on the physics of compact, strongly gravitating objects such as neutron stars (NSs) and black holes (BHs), providing information complementary to that available from electromagnetic and neutrino observations, and plausibly producing important insights into unsolved questions such as the equation of state (EOS) of matter at nuclear densities, and the mechanisms behind gamma-ray bursts. Detailed reviews of GW sources, of the expected event rates, and of the physics that these sources could teach us are available elsewhere,^{1,2,3} but I shall list briefly the most promising astrophysical systems from which we could learn about NSs using Earth-based GW interferometers such as LIGO and VIRGO.

1. *NS-NS and NS-BH binaries in the last few minutes of their inspirals.* For a long time, these inspiraling systems have been the prototype for the category of the short-lived *chirp* signals detectable using Earth-based interferometers. The reason, of course, is that NS-NS binaries have actually been observed in our galaxy,⁴ but also that the part of the inspiral accessible to the interferometers (with GW frequencies between 40 and 1000 Hz) sits well before the final merger of the binary, so it is described very accurately by the well-developed post-Newtonian equations for point masses.⁵ The successful observation of GWs from these events will teach us about the masses, spins and locations of NSs, but not about their internal structure.

On the contrary, the detection of GW from the endpoint of NS-BH inspirals should produce detailed information about NS structure and EOS. For a wide range of binary parameters, the NS will be torn apart by the tidal field of the BH well before the final plunge into the hole, and the tidal-

disruption waves will be well inside the frequency range of good interferometer sensitivity.⁶ NS-BH binaries have also been proposed as engines for gamma-ray bursts⁷ and as suitable environments for the production of heavy nuclei in *r*-processes.⁸ I will discuss these systems more extensively in Sec. II of this paper.

2. *Rapidly spinning, deformed NSs.* This class includes the known and unknown radio pulsars (when their gravitational ellipticity is high enough to provide strong GWs), and the systems known as low-mass X-ray binaries (LMXB), where the NS is accreting matter and angular momentum from a companion, but instead of increasing its rotation, it is locked into spin periods of about 3 ms; it is conjectured that the angular momentum being accreted is lost to the emission of GW.⁹

To detect rapidly spinning NSs, it will be necessary to integrate the GW signal for times up to several months, so the Doppler frequency modulation caused by the movement of the Earth around the sun will make it much harder to detect previously unknown sources.¹⁰ At the same time, the shape of this modulation will make it possible to obtain the position of the source in the sky,¹¹ and to match the GW source with one of the objects known from electromagnetic observations.

If any GWs are detected, their features would be very informative, in particular when examined in correlation with electromagnetic signals from the same source. For instance, the ratio of the GW frequency to the NS angular frequency could identify the nature of the inhomogeneities that give rise to the GW emission, and the evolution of the GW amplitude and frequency could provide interesting data about NS physics such as crust structure and dynamics, crust-core interactions, magnetic fields, viscosity, superfluidity, and more.³

3. *Proto-neutron stars.* Finally, NSs could be observed as the rapidly spinning, strongly asymmetric remnants of stellar-core collapse, or as the proto-NSs produced by the accretion-induced collapse of white dwarves. Proto-NSs that spin very fast can

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eter sensitivity for the advanced interferometers such as LIGO-II. It follows that, in principle, we could use the waveforms from a NS tidal-disruption event to measure both the NS mass and the NS radius. However, we still need to know just how well we could measure them.

B. The matched-filtering paradigm for gravity-wave data analysis

To answer this question, we need to invoke the theory of *matched-filtering parameter estimation*.¹⁷ The general idea is that GWs will be detected by correlating the measured signal $|s\rangle$ to a bank of theoretical *templates* $\{|h_i\rangle\}$, which represent our best approximation of the realistic GW signal, as it could happen for a variety of binary parameters. If the *match* $\langle s|h_i\rangle$ (the correlation between $|h_i\rangle$ and $|s\rangle$) is much higher than the match $\langle h_i|n\rangle$ that the template would give, on the average, with noise alone, then we claim that we have a detection. To know how well we can estimate R , we ask how probable it is that a particular realization of detector noise would lead us to mistake the template $|h_R\rangle$ with the nearby template $|h_{R+\Delta R}\rangle$: the answer is given in terms of the match $\langle h_R|h_{R+\Delta R}\rangle$.

To compute this match we construct a bank of signal templates that differ only⁴² in the GW frequency at the onset of tidal disruption. Because we are not interested in modeling accurately the relativistic dynamics of the binary, but only the effects of tidal disruption, we generate our waveforms from simple quadrupole-governed Newtonian inspirals,¹⁸ cutting off the signal, more or less abruptly,⁴³ when the instantaneous GW frequency reaches f_{td} . Computing the match between nearby templates, we estimate the granularity to which f_{td} can be measured for a given signal strength (inversely proportional to distance); we can then use our equation for $f(R)$ to propagate the errors to the NS radius. The final result of this exercise is that, employing advanced Earth-based interferometers such as LIGO-II³, we should be able to measure the NS radius to 15%, for tidal-disruption events at distances that yield about one event per year.⁶ This estimated error should be compared to the error (about a factor of two) in the measurements of the NS *thermal radius*.

C. Inverting the mass–radius relation

What can we do once we get a few points on the $m(R)$ curve? We can try to solve for the EOS of dense nuclear matter in NSs. The relativistic model of nonrotating NSs may be considered as a mapping from the NS EOS, through the Oppenheimer–Volkoff (OV) equations,

$$\frac{dm}{dr} = 4\pi r^2 \rho, \quad \frac{d\rho}{dr} = -(\rho + p) \frac{m + 4\pi r^3 p}{r(r - 2m)}, \quad (1)$$

to macroscopic NS quantities such as mass and radius. Let us work through this mapping in the case of a polytropic EOS, $p = p(\rho)$. First, we set the central density ρ_c ; then, we solve the OV equations and compute the NS radius $R = R(\rho_c)$ and mass $m = m(\rho_c)$; finally, we eliminate ρ_c from these two equations, completing our mapping of the EOS $p(\rho)$ into the mass–radius relation $m(R)$.

Lindblom²⁰ has shown how to invert the OV mapping using even a few $m(R)$ data points. Suppose that $p(\rho)$ is known up to a certain density ρ_{max} from other observations and experiments. Now start with the least dense NS, and integrate the OV equations backward, from the surface of the star [where we know R and $m(R)$] down to the radius where $\rho = \rho_{\text{max}}$. We are then left with a stellar core of known mass and radius, and we can use an analytic approximation for the solution of the OV equations to get p_c and ρ_c ; we add the point $p_c(\rho_c)$ to the EOS, and repeat for the next NS. The result is $p(\rho)$ as a piecewise linear law. This analysis can be generalized to include rotationally deformed models, and to account for the statistical uncertainty in the $m(R)$ data. Recently, Harada²¹ has shown how other macroscopic parameters of NSs (such as moments of inertia, baryonic masses, binding energies, gravitational redshifts) can be used in this same framework to recover information about the EOS.

Finally, Saijo and Nakamura²² have suggested that it might be possible to measure the NS radius directly from the spectrum of the GWs emitted in NS–BH coalescences. These authors have used BH perturbation theory to compute the spectrum of GW emitted by a disk of dust inspiraling into a rotating BH. When the radius R_{disk} is larger than the wavelength of the quasi-normal modes of the BH, the spectrum acquires several peaks with separation $\propto R_{\text{disk}}^{-1}$, irrespective of M and a . Saijo and Nakamura conjecture that the same structure would be visible in the spectrum of GW signals from NS–BH binaries, providing direct information about the radius. However, several issues are left unaddressed: in particular, the particles of the disk move along geodesics, while the fluid of a NS would be strongly constrained by the gravitation and pressure of the star (except, perhaps, in the regime of severe tidal disruption); furthermore, for coalescence events that happen at realistic distances the signal strength might be too low to let us resolve the form-factor structure in the spectrum.

D. Numerical simulations of neutron-star tidal disruption

Summarizing, the preliminary analysis carried out by the present author suggests that NS tidal-disruption events have much to teach us about the NS EOS. This prediction is confirmed by Newtonian simulations²³ of NS–BH systems, carried out using both smooth particle hydrodynamics^{8,24} and Eulerian techniques^{7,25}, which

show that the ultimate fate of the system (complete or incomplete disruption; presence of accretion rings or tidal tails; features of the GW shutoff) depends strongly on the stiffness of the EOS. However, detailed relativistic numerical simulations are still needed to confirm these prospects, and will be essential as a foundation to interpret any tidal-disruption waveforms that might be measured in reality.

III. GRAVITY WAVES FROM THE r -MODES OF YOUNG NEUTRON STARS

All rotating stars possess a class of circulation modes (r -modes) that are driven toward instability by gravitational radiation reaction; in hot, rapidly rotating young NSs, this destabilizing effect might be so strong that it dominates viscous dissipation. Once an r -mode achieves sufficient amplitude, the star is quickly spun down as angular momentum is lost to gravitational radiation.^{13,14} For stars with initial angular velocity $\Omega \sim 1000$ Hz, the timescale for r -mode growth is ~ 40 s. In recent years, r -modes have attracted considerable interest as a possible explanation for the failure to observe very rapidly spinning pulsars, and as a promising GW source for Earth-based detectors.

However, the astrophysical relevance of r -modes is still in doubt, pending judgment on two separate issues. First, the instability of r -modes (discovered by analyzing the linearized Euler equations for perfect fluids) might not be confirmed after all the complicated physics that occurs in NSs is taken into account, including relativistic effects, physical-EOS effects, solid crust effects, magnetic fields, rotation laws, and so on. Second, the amount of angular momentum removed from the star and the strength of the GW radiation emitted depend critically on the maximum amplitude that can be reached by the r -mode; but the growth of the r -mode might be limited to a very small *saturation amplitude* by the effects of magnetic fields²⁶, or by the leakage of energy to other (damped) modes by way of nonlinear hydrodynamical couplings.

There have been several attempts to investigate the nonlinear dynamics of the r -modes, by means of second-order Lagrangian perturbation theory,²⁷ and of relativistic numerical simulations in the Cowling approximation.²⁸ In addition, Lindblom, Tohline and the present author have carried out Newtonian numerical simulations that include radiation reaction as an effective force. We now briefly review this work, but we invite the reader to refer to the original articles^{29,30} for further details.

A. Numerical evolutions of nonlinear r -modes in neutron stars

We solved the Newtonian Euler and Poisson equations on a $128 \times 64 \times 128$ cylindrical grid, using a 2nd-

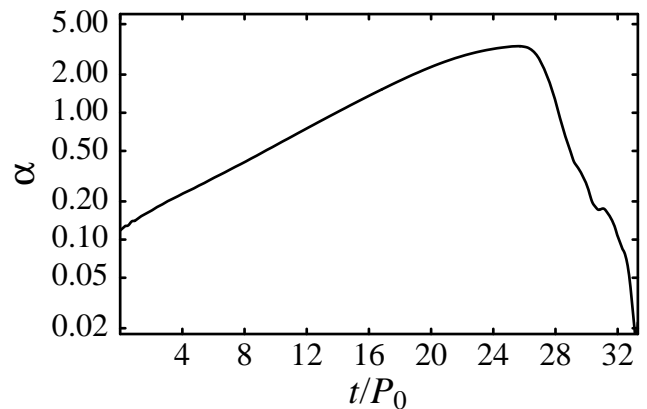


FIG. 2: Evolution of the dimensionless r -mode amplitude α . Amplitudes of order one imply mode velocities comparable to the local velocity of rigid rotation. Time is measured in units of the initial rotation period P_0 of the NS.

order-accurate finite-difference code developed at LSU³¹ to tackle a variety of astrophysical problems. The LSU code was parallelized³² in the domain-decomposition paradigm using the well known MPI³³ library. We ran it on 16 nodes of the HP V-2500 supercomputers at Caltech's Center for Advanced Computing Research.³⁴

Our initial equilibrium configuration was a simple polytrope (with $n = 1$, $M = 1.4 M_\odot$, $R = 12.5$ km) obtained by solving selfconsistently the Bernoulli and Poisson equations.³⁵ We then augmented the rigid-rotation velocity field with a small-amplitude, slow-rotation approximation to an $l = m = 2$ r -mode,³⁶

$$\delta \vec{v} = \alpha_0 R \Omega_0 \left(\frac{r}{R} \right)^2 \vec{Y}_{22}^B e^{i\omega_0 t}, \quad (2)$$

where R and Ω_0 are the radius and angular velocity of the unperturbed star, α_0 is the dimensionless r -mode amplitude, and \vec{Y}_{22}^B is a vector spherical harmonic of the magnetic type. Gravitational radiation reaction was added as an effective Newtonian force³⁷ proportional to the derivatives of the current multipole moments of the star.

Our computing budget set tight limits on the length of the simulation: one full rotation period of a rapidly rotating NS took up to ten thousand CPU hours. Consequently, we could not afford to follow the growth of the r -mode through one or more radiation timescales. Instead, we increased the strength of the effective radiation-reaction force by a factor of about 4500, to bring down the r -mode growth timescale to values comparable to the rotation period. Even with this trick, we expect that the physical behavior observed in the simulations should still be realistic, because the nonlinear dynamics of the r -mode (including its couplings to other modes) should happen at the hydrodynamical timescale, which in our simulations is still much shorter than the radiation-reaction timescale.

While computing the evolution of the star, we monitored the r -mode amplitude α by projecting the velocity

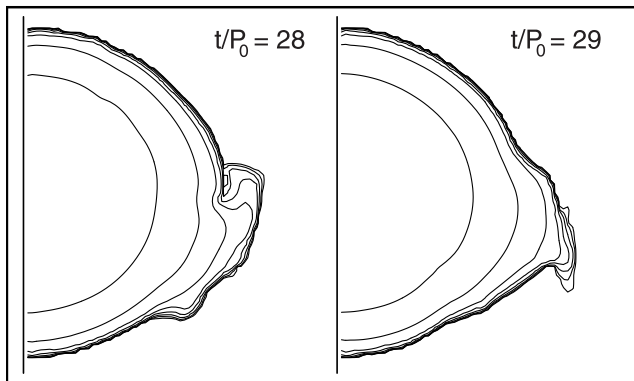


FIG. 3: Cresting waves at the surface of the NS, after the r -mode amplitude has reached its maximum. The waves create strong shocks that dump mode energy into thermal energy.

of the fluid on \vec{Y}_{lm}^B ,

$$\alpha = \frac{2|J_{22}|}{(\Omega/R) \int \rho r^6 dr}, \quad (3)$$

where J_{22} is the $l = m = 2$ current multipole moment; similarly, we read off the mode frequency ω as

$$\omega = -\frac{1}{|J_{22}|} \left| \frac{dJ_{22}}{dt} \right|. \quad (4)$$

At the beginning of our evolution, these diagnostics recovered the theoretical values α_0 and ω_0 within the expected error.

Figure 2 shows the evolution of the r -mode amplitude. For this simulation, the theoretical prediction³⁶ for the r -mode growth time was 10 initial rotation periods. The mode grew exponentially (in good accord with theory) until $\alpha \sim 2$; then the mode started to be limited by some nonlinear process, the amplitude peaked at $\alpha = 3.35$, and finally fell down very rapidly. A movie of the crashing r -mode can be found on the web.³⁸

What nonlinear process was responsible for limiting the growth of the r -mode, and for causing its rapid demise? We discovered a clue when we examined the evolution of the energy and angular momentum in the course of the simulation. Even after the emission of angular momentum into GWs fell to zero, the star continued to lose energy; something other than GWs must be responsible for this loss.

We believe that we have found the culprit. To first order in the amplitude, the r -mode is only a *velocity* mode; to second order, however, there is an associated density perturbation, proportional to Y_{32} , which appears as a wave with four crests (two in each hemisphere) on the surface of the star. As the amplitude reaches its maximum, these propagating crests turn into large, breaking waves (see Fig. 3), and the edges of the waves develop strong shocks that dump kinetic energy into thermal energy, killing the r -mode.

B. Numerical simulations as a laboratory of stellar physics

Our code provided a nice laboratory to perform several more evolutions and tests.

1. We performed basic tests on the robustness of the code and of our diagnostics, evolving stars with different angular velocities, with or without r -mode perturbations, and at different grid resolutions.
2. We investigated the dependence of the saturation amplitude on the artificial amplification of radiation reaction. Our computing budget made it impossible to increase the radiation-reaction timescale; instead, we reduced it even more, finding that the r -mode saturates faster, but at essentially the same amplitude.
3. We studied the *unforced* evolution of unit-amplitude r -modes, finding that they are essentially stable for as long as we could evolve them. These results are compatible with the relativistic evolutions (also unforced) performed by Stergioulas and Font.²⁸

C. A new picture for gravity-wave signals from r -modes

In the traditional scenario for GW emission from r -modes,³⁹ the unstable r -mode would grow until it reached a dimensionless amplitude of about one, and then it would saturate and persist at that amplitude (for several months) until it would have lost most of its angular momentum; during that time, the frequency of the r -mode would decrease in proportion with the NS spin. The prospective picture of r -mode GW signals that emerges from our evolutions is quite different. Most interesting, the r -mode spindown episodes are faster (only a few minutes), and the GW frequency remains remarkably constant as the angular velocity of the star decreases. As a result, the search for r -mode signals changes from a pulsar-like search (which must account for the Doppler shifts generated by the movement of the Earth around the sun) to an easier chirp-like search, with encouraging prospects for detection.⁴⁰

Although recent studies of the effect of magnetic fields,²⁶ and of exotic forms of bulk viscosity⁴¹ suggest that the astrophysical significance of r -modes might be limited, the considerable uncertainty about the macroscopic and microscopic structure and dynamics of NSs makes it reasonable to devote some resources to GW searches for r -mode signals similar to those predicted by the simulations described above.

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- ¹ V. Ferrari, proceedings of the 25th J. Hopkins Workshop on Current Problems in Particle Theory. 2001: A Relativistic Spacetime Odyssey, Florence, Sep. 3–5, 2001. [astro-ph/0201265](#).
 - ² S. A. Hughes, S. Márka, P. L. Bender, and C. J. Hogan, proceedings of the 2001 Snowmass Meeting, Snowmass Village, Colorado, June 30–July 21, 2001. [astro-ph/0110349](#).
 - ³ K. S. Thorne, “The Scientific Case for Advanced LIGO Interferometers,” LIGO Doc. P-000024-00-D (2001).
 - ⁴ See for instance S. E. Thorsett, and D. Chakrabarty, *Astrophys. J.* **512**, 288 (1999).
 - ⁵ See for instance L. Blanchet, proceedings of the 16th International Conference on General Relativity and Gravitation, Durban, South Africa, July 15–21, 2001. [gr-qc/0201050](#).
 - ⁶ M. Vallisneri, *Phys. Rev. Lett.* **84**, 3519 (2000).
 - ⁷ H.-T. Janka, T. Eberl, M. Ruffert, and C. Fryer, *Astrophys. J.* **527**, L39 (1999).
 - ⁸ W. Lee, *MNRAS* **318**, 606 (2000).
 - ⁹ L. Bildsten, *Astrophys. J. Lett.* **501**, L89 (1998); G. Ushomirsky, C. Cutler, and L. Bildsten, *MNRAS* **319**, 902 (2000), and references therein.
 - ¹⁰ P. R. Brady, T. Creighton, C. Cutler, and B. F. Schutz, *Phys. Rev. D* **57**, 2101 (1998).
 - ¹¹ P. Jaranowski, and A. Krolak, *Phys. Rev. D* **59**, 063003 (1999).
 - ¹² See for instance M. Shibata, T. W. Baumgarte, and S. L. Shapiro, *Astrophys. J.* **542**, 453 (2000).
 - ¹³ For a recent review, see L. Lindblom, in *Gravitational Waves: A Challenge to Theoretical Astrophysics*, V. Ferrari, J. C. Miller, and L. Rezzolla, eds. (ICTP, Trieste, 2001). [astro-ph/0101136](#).
 - ¹⁴ K. D. Kokkotas, proceedings of the 25th J. Hopkins Workshop on Current Problems in Particle Theory. 2001: A Relativistic Spacetime Odyssey, Florence, Sep. 3–5, 2001.
 - ¹⁵ K. S. Thorne, in *Three hundred years of gravitation*, S. Hawking and W. Israel, eds. (Cambridge, Cambridge University Press, 1987), p. 330; A. Abramovici, *et al.*, *Science* **256**, 325 (1992).
 - ¹⁶ S. Chandrasekhar, *Ellipsoidal figures of equilibrium* (Yale University Press, New Haven, 1969); M. Shibata, *Prog. Theor. Phys.* **96**, 917 (1996); P. Wiggins, and D. Lai, *Astrophys. J.* **532**, 530 (2000).
 - ¹⁷ See for instance L. S. Finn, *Phys. Rev. D* **46**, 5236 (1992).
 - ¹⁸ See for instance C. W. Misner, K. S. Thorne, and J. A. Wheeler, *Gravitation* (W. H. Freeman and Co., San Francisco, 1973).
 - ¹⁹ L. Bildsten, and C. Cutler, *Astrophys. J.* **400**, 175 (1992).
 - ²⁰ L. Lindblom, *Astrophys. J.* **398**, 569 (1992).
 - ²¹ T. Harada, *Phys. Rev. C* **64**, 048801 (2001).
 - ²² M. Saijo, and T. Nakamura, *Phys. Rev. D* **63**, 064004 (2001); *Phys. Rev. Lett.* **85**, 2665 (2001).
 - ²³ For a review of numerical simulations of GW sources, including references to NS–NS and NS–BH simulations, see J. M. Centrella, proceedings the Conference on Stellar Collisions, American Museum of Natural History, June 2000. [gr-qc/0011109](#)
 - ²⁴ W. Lee, and W. Kluźniak, *Acta Astron.* **45**, 705 (1995); *Astrophys. J. Lett.* **494**, L53 (1998); *Astrophys. J.* **526**, 178 (1999); *MNRAS* **308**, 780 (1999); *MNRAS* **328**, 583 (2001).
 - ²⁵ H.-T. Janka, and M. Ruffert, in *Proceedings of the 10th workshop on “Nuclear Astrophysics”*, W. Hillebrandt and E. Müller, eds. (Garching, Germany, Max-Planck-Institut für Astrophysik, 2000), p. 136.
 - ²⁶ L. Rezzolla, F. L. Lamb, D. Markovic, and S. L. Shapiro, *Phys. Rev. D* **64**, 104013 (2001); *Phys. Rev. D* **64**, 104014 (2001).
 - ²⁷ A. K. Schenk, P. Arras, É. É. Flanagan, S. A. Teukolsky, and Ira Wasserman, *Phys. Rev. D* **65**, 024001 (2002).
 - ²⁸ N. Stergioulas and J. A. Font, *Phys. Rev. Lett.* **86**, 1148 (2001).
 - ²⁹ L. Lindblom, J. Tohline and M. Vallisneri, *Phys. Rev. Lett.* **86**, 1152 (2001).
 - ³⁰ L. Lindblom, J. Tohline and M. Vallisneri, *Phys. Rev. D*, in press (2002). [astro-ph/0109352](#).
 - ³¹ K. C. B. New and J. E. Tohline, *Astrophys. J.* **490**, 311 (1997); H. S. Cohl and J. E. Tohline, *Astrophys. J.* **527**, 527 (1999); J. E. Cazes and J. E. Tohline, *Astrophys. J.* **532**, 1051 (2000).
 - ³² P. M. Motl, J. Tohline, and J. Frank, *Astrophys. J. Suppl. Ser.* **138**, 121 (2002).
 - ³³ MPI forum website, [www.mpi-forum.org](#).
 - ³⁴ CACR website, [www.cacr.caltech.edu](#).
 - ³⁵ I. Hachisu, *Astrophys. J. Suppl. Ser.* **61**, 479 (1986); *Astrophys. J. Suppl. Ser.* **62**, 461 (1986).
 - ³⁶ L. Lindblom, B. J. Owen, and S. M. Morsink, *Phys. Rev. Lett.* **80**, 4843 (1998).
 - ³⁷ L. Rezzolla, M. Shibata, H. Asada, T. W. Baumgarte, and S. L. Shapiro, *Astrophys. J.* **525**, 935 (1999).
 - ³⁸ Michele Vallisneri’s *r*-mode webpage, including movies of the instability, [www.vallis.org/rmode](#).
 - ³⁹ B. J. Owen, L. Lindblom, C. Cutler, B. F. Schutz, A. Vecchio, and N. Andersson, *Phys. Rev. D* **58**, 084020 (1998).
 - ⁴⁰ B. J. Owen and L. Lindblom, proceedings, Fourth Amaldi Conference on Gravitational Waves, July 2001, Perth. [gr-qc/0111024](#).
 - ⁴¹ B. J. Owen and L. Lindblom, *Phys. Rev. D*, in press (2002). [astro-ph/0110558](#).
 - ⁴² In the realistic case, the templates depend on *all* the binary parameters. The estimation problem then becomes involved, because there can be correlations in the ways that different parameters modify the templates. In our simple model, we assume that all parameters except f_{td} are already known well from the early part of the inspiral signal, so all that is left to do is to find f_{td} .
 - ⁴³ Bildsten and Cutler¹⁹ estimate that complete disruption would take place in ~ 1 –3 orbital periods, while the disrupted NS would spread into a ring in ~ 1 –2 periods, significantly reducing the GW amplitude.